Extensions to Maxwell's Equations

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Abstract

The development of Maxwell's traditional equations based on the common field approach is presented in this paper. The generalized equation for electromagnetic transformations is found. The extensions to Maxwell's traditional equations are obtained.

I. Introduction

Maxwell's equations are the base of classical electrodynamics [1]. It connects the transformation of the electrical field to the magnetic field and vice versa through their intensities **E** and **B,** and also determines their connection with the source through the charge density ρ and the current densities **j**:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{(1a)} \qquad \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \quad \text{(1b)}$$

$$\nabla \times \vec{B} = \frac{\vec{j}}{c^2 \cdot \varepsilon_0} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t} \quad \text{(1c)} \qquad \nabla \cdot \vec{B} = 0 \quad \text{(1d)} \qquad (1)$$

Maxwell's equations describe the vectors fields: the vector **B** contains only one component as a rotational field, although the vector **E** is a hybrid field containing irrotational and rotational components. The equations are not symmetrical: the right side is the cause, and the left side is the consequence of it. However, the transformation principle of one field to another is not visible in it, and, as a result, the connection of the equations' sides is not obvious. This fact pushed the author to figure out another way to present the fundamental equations of electrodynamics.

II. Generalized Equation

It is well known that there are three types of vector fields such as gradient, rotational and hybrid fields. According to Helmholtz's theorem, any sufficiently smooth, rapidly decaying vector field in three dimensions can be resolved into the sum of the vector fields: irrotational (gradient) field and a solenoidal (rotational) field.

For analysis generality, it is assumed that initial vector's field **h** is hybrid, i.e. it may be presented as the combination of gradient **g** and rotational **r** fields:

$$h = g + r \tag{2}$$

Let’s assume that transformation of the initial field to another kind of field is occurring through the circulation process described by rotor (curl) operator:

$$r' = \nabla \times h \tag{3}$$

The conversed field **r’** is a rotational field, so it makes sense to determine its characterization as the vorticity:

$$\nabla \times r' = \nabla \times (\nabla \times h) \tag{4}$$

As shown in [2], the right side of (4) can be presented as follows:

$$\nabla \times (\nabla \times a) = \nabla (\nabla \cdot a) - \nabla^2 a \tag{5}$$

With the formula (5), the equation (4) changes to:

$$\nabla \times r' = \nabla (\nabla \cdot h) - \nabla^2 h \tag{6}$$

Taking into account (1), i.e. the nature of initial field, the expression (5) takes the next form:

$$\nabla \times r' = \nabla (\nabla \cdot g) - \nabla^2 h \tag{7}$$

The obtained equation establishes the relation between initial and transformed fields through the rotor operator. The rotor of the transformed field is the combination of the two vectors and one of them has a gradient nature. It should be noticed that the physical nature of the fields involved in the transformation process is not stipulated, but it is assumed that the physical nature of transformed field **r’** is different from the initial **h** one. The circulation process is the base for the transformation of different kinds of fields in nature, so it looks like this process is fundamental. In this case, the obtained equation (7) may be considered as a generalized equation for the transformation of vector fields corresponding to the process (3).
Thus, for electrodynamics, the generalized equation (7) is the base for the transformation of electrical field to magnetic field and vice versa.

III. Equations’ Extensions

To prove it, the common designations of the fields in (7) should be substituted to their physical parameters. The initial hybrid field **h** may be replaced by vector of electrical field intensity $\mathbf{E}^h = \mathbf{h}$, which is also hybrid. The scalar value under gradient operator in (7) may be substituted by the well-known expression $\nabla \cdot E^g = \rho / \varepsilon_0$. Based on the last expression, the equation (7) changes to:

$$\nabla \times r' = \nabla\left(\frac{\rho}{\varepsilon_0}\right) - \nabla^2 E^h \tag{8}$$

All operators in (8) are spaced, although the conversion (3) is occurring in time. The transition to the timing ratios may be performed formally. The gradient operator in (8) is the fact that the charge transfer is occurring. Let the charge transfer run on the x-coordinate in the Cartesian reference system:

$$\nabla_x\left(\frac{\rho}{\varepsilon_0}\right) = \frac{1}{\varepsilon_0}\frac{\partial \rho}{\partial x}\vec{e}_x = \frac{1}{\varepsilon_0}\frac{\partial \rho}{\partial t}\frac{\partial t}{\partial x}\vec{e}_x = \frac{1}{\varepsilon_0\left|\vec{\vartheta}_x\right|}\frac{\partial \rho}{\partial t}\vec{e}_x \tag{9}$$

where $\vec{\vartheta}_x$ is the speed of charge transfer on the x-coordinate; $\vec{e}_x$ is the unit vector.

Taking into account [3] and electron's nature of the current, the (9) is transferred in the following:

$$\nabla_x\left(\frac{\rho}{\varepsilon_0}\right) = -\frac{1}{\varepsilon_0 \vartheta_x^2}\frac{\partial \vec{j}_x}{\partial t} \tag{10}$$

The Laplasian operator (8) may also be converted to the timing ratios. With the presentation of the x-component, as shown in [4], it may be converted by approach (9) in the following:

$$(\nabla^2 E^h)_x = \left(\frac{\partial^2 E_x^h}{\partial t^2}\frac{1}{\left|\vec{v}_x\right|^2} + \frac{\partial^2 E_y^h}{\partial t^2}\frac{1}{\left|\vec{v}_y\right|^2} + \frac{\partial^2 E_z^h}{\partial t^2}\frac{1}{\left|\vec{v}_z\right|^2}\right)\vec{e}_x \tag{11}$$

$\vec{v}_x, \vec{v}_y, \vec{v}_z$ are the speed on coordinates x, y, z.

The similar transformations under the same condition may be performed for the left side of (8) with the rotor operator. Taking into account the nature of **E** it gives the next:

$$(\nabla \times E^r)_x = \left(\frac{\partial E_z^r}{\partial y} - \frac{\partial E_y^r}{\partial z}\right)\vec{e}_x = -\left(\frac{1}{\left|\vec{v}_y\right|}\frac{\partial E_z^r}{\partial t} - \frac{1}{\left|\vec{v}_z\right|}\frac{\partial E_y^r}{\partial t}\right)\vec{e}_x \tag{12}$$

where $\vec{v}_y = -\left|\vec{v}_y\right|\vec{e}_y, \vec{v}_z = -\left|\vec{v}_z\right|\vec{e}_z$.

In case that the space is an isotropic, the above-all mentioned speeds are satisfied to the condition $\left|\vec{v}_x\right| = \left|\vec{v}_y\right| = \left|\vec{v}_z\right| = \left|\vec{\vartheta}_x\right| = c$, where c is the light speed.

Based on (10)-(12) and after the expanding their expressions to all coordinates, the formula (8) takes the form:

$$\nabla \times \frac{1}{c}\frac{\partial r'}{\partial t} = -\frac{1}{c^2 \cdot \varepsilon_0}\frac{\partial \vec{j}}{\partial t} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\left(\vec{E}_x^h + \vec{E}_y^h + \vec{E}_z^h\right) \quad (13)$$

where $r' = -\left((E_z^r - E_y^r)\vec{e}_x + (E_x^r - E_z^r)\vec{e}_y + (E_y^r - E_x^r)\vec{e}_z\right)$.

The rotor operator in (13) affects the vector that is orthogonal to the vector $\mathbf{E}^h$ and has another physical nature and amplitude. In this case, the transition to magnetic field intensity with amplitude $|\mathbf{B}| = |\mathbf{E}^h|/c$ is quite fair, i.e.:

$$-\nabla \times \frac{\partial \vec{B}}{\partial t} = -\frac{1}{c^2 \cdot \varepsilon_0}\frac{\partial \vec{j}}{\partial t} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\left(\vec{E}_x^h + \vec{E}_y^h + \vec{E}_z^h\right) \quad (14)$$

It should be noticed that the last term in the right side of (14) takes the form of the wave equation. The expression (14) may be simplified by reducing the derivation order:

$$\nabla \times \vec{B} = \frac{\vec{j}}{c^2 \cdot \varepsilon_0} + \frac{1}{c^2}\frac{\partial}{\partial t}\left(\vec{E}_x^h + \vec{E}_y^h + \vec{E}_z^h\right) \quad (15)$$

In general, the obtained equation corresponds to the traditional form of equation (1c) except that the hybrid field $\mathbf{E}^h$ is presented by the extension as the sum of its all coordinate projections.
The similar procedure can be performed for the inverted transformation of the magnetic field to the electric field. In this case, the initial field is presented by the rotational field **r** only. Taking into account the feature of rotational (curl-free) field, the equation (7) takes the form of:

$$\nabla \times r' = -\nabla^2 r \quad (16)$$

The operations similar to (11) and (12) may be applied, and formula (16) becomes as follows:

$$\nabla \times \frac{1}{c}\frac{\partial r'}{\partial t} = -\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\left(\vec{B}_x + \vec{B}_y + \vec{B}_z\right) \quad (17)$$

The rotor operator in (17) affects the vector that is orthogonal to the vector **B** and has another physical nature and amplitude. In this case, the transition to the electric field intensity with amplitude $|\mathbf{E}| = c\cdot|\mathbf{B}|$ is fair. The expression (17) may be simplified by reducing the derivation order, and it eventually takes the form:

$$\nabla \times \vec{E} = -\frac{\partial}{\partial t}\left(\vec{B}_x + \vec{B}_y + \vec{B}_z\right) \quad (18)$$

As well as equation (15), the equation (18) in general corresponds to the traditional form of equation (1a), except that the rotational field **B** is presented by the extension as the sum of its all coordinate projections.

The obtained extensions (15) and (18) may be united to be considered as Maxwell's extended equations:

$$\nabla \times \vec{B} = \frac{\vec{j}}{c^2 \cdot \varepsilon_0} + \frac{1}{c^2}\frac{\partial}{\partial t}\left(\vec{E}_x^h + \vec{E}_y^h + \vec{E}_z^h\right)$$
$$\nabla \times \vec{E} = -\frac{\partial}{\partial t}\left(\vec{B}_x + \vec{B}_y + \vec{B}_z\right) \tag{19}$$

The extended equations (19) allow reconsidering the problems of electrodynamics that is already solved based on the Maxwell's traditional equations (1). It should be noticed, that the formulas (1b) and (1d) are presented in (1) as independent equations, but according to the present development they are used just to establish the structure of the Maxwell's extended equations (19). In addition, Maxwell's equation (1a) is interpreted as Faraday's law. If the obtained extensions are correct, then Faraday's law may be reconsidered based on equation (18). For example, the well known exception from the "flow rule" [4], it may now be explained based on (18) without any exemption. Also, there are mismatches [5] for some of electrodynamics applications based on traditional equations. The obtained extensions, probably, allow to eliminate the existing mismatches and to see in a new light the well-known applications that have been already obtained by Maxwell's traditional equations.

IV. Conclusion

Thus, the equation (7) may be considered as the generalized field equation for the electromagnetic transformations. Maxwell's traditional equations can be obtained from (7) based on the direct substitution of corresponding physical parameters of electrical and magnetic fields. The obtained extended equations (19) allow reconsidering well-known applications of Maxwell's traditional equations and possibly eliminating the existing mismatches.

Acknowledgement

The author does not have any opportunity to perform experimental researches in a laboratory environment, but he hopes that other researchers will obtain experimental confirmation of the proposed extensions and thanks them in advance.

References

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